Talk 1: Economic & Statistical Concerns in the Design of Multivariate Control Charts

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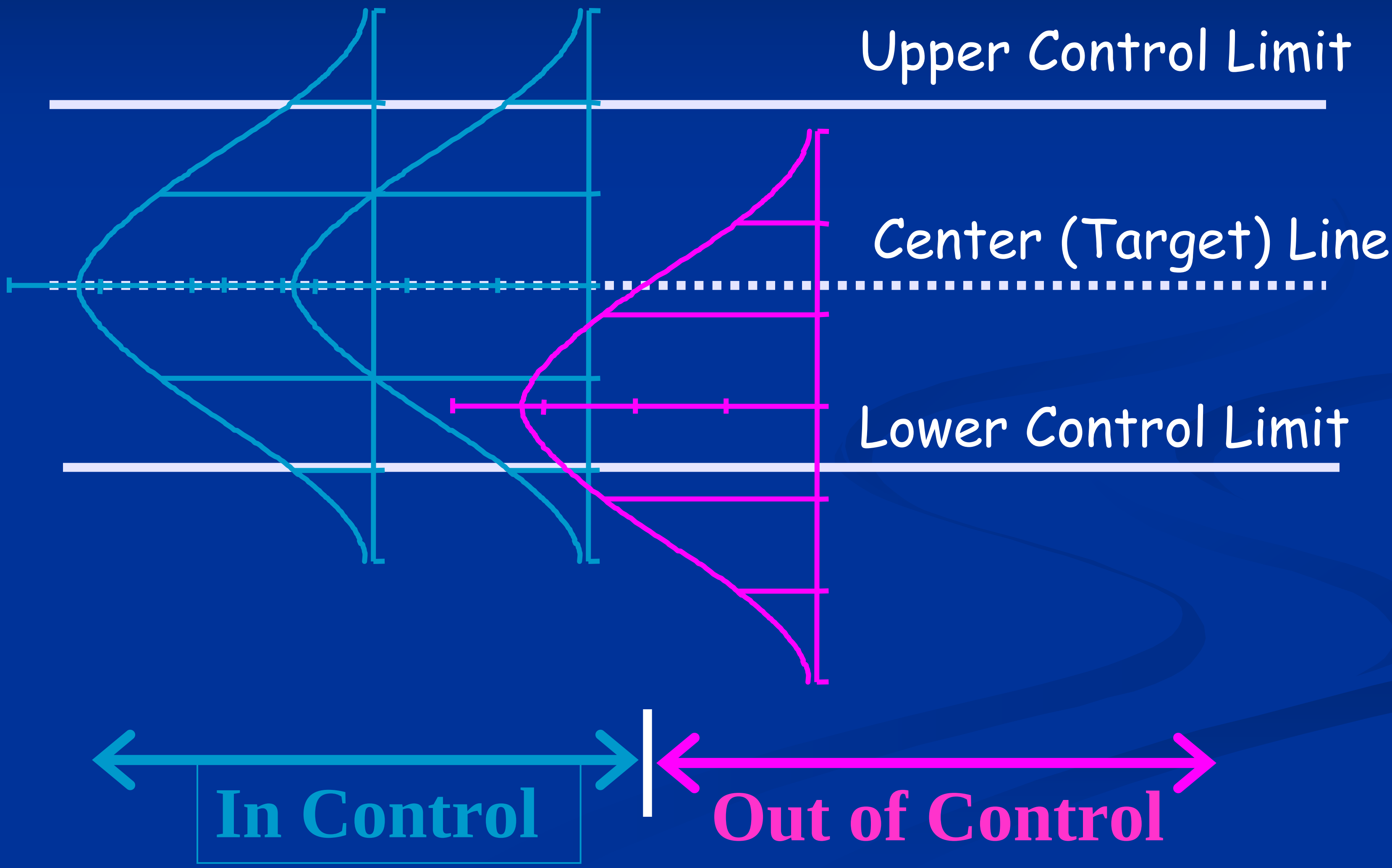
Statistical Process Control Goals

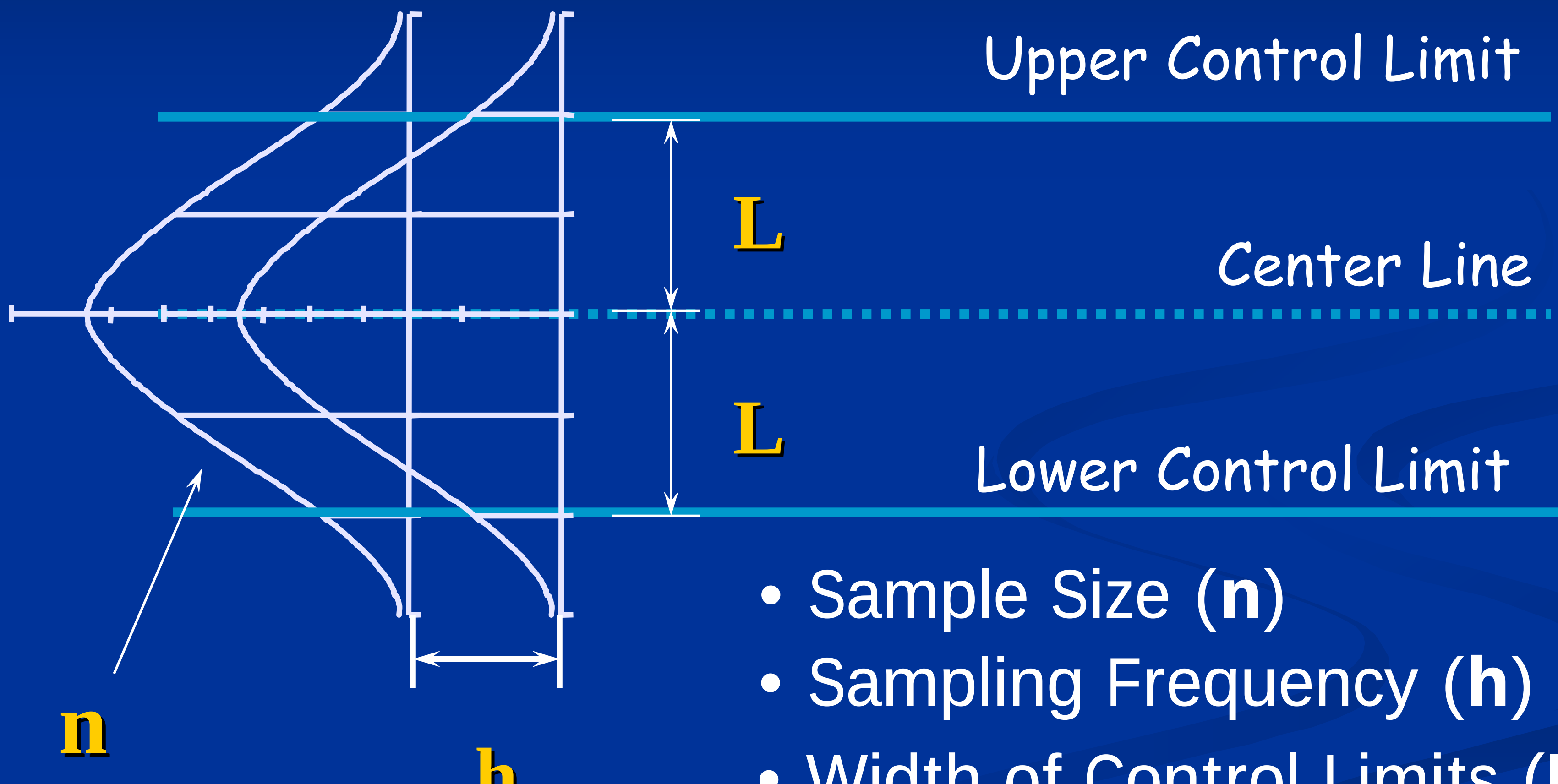
(Czitrom & Spagon, (1997). *Statistical Case Studies for Industrial Process Improvement, Chapter 21)*

- Monitor and reduce process variability
- Monitor and maintain the process on target -- establish process stability
- Determine when the process needs adjusting and when it doesn’t
- Detect process changes to enable corrective action

Control Charts



Design Parameters for Shewhart Charts



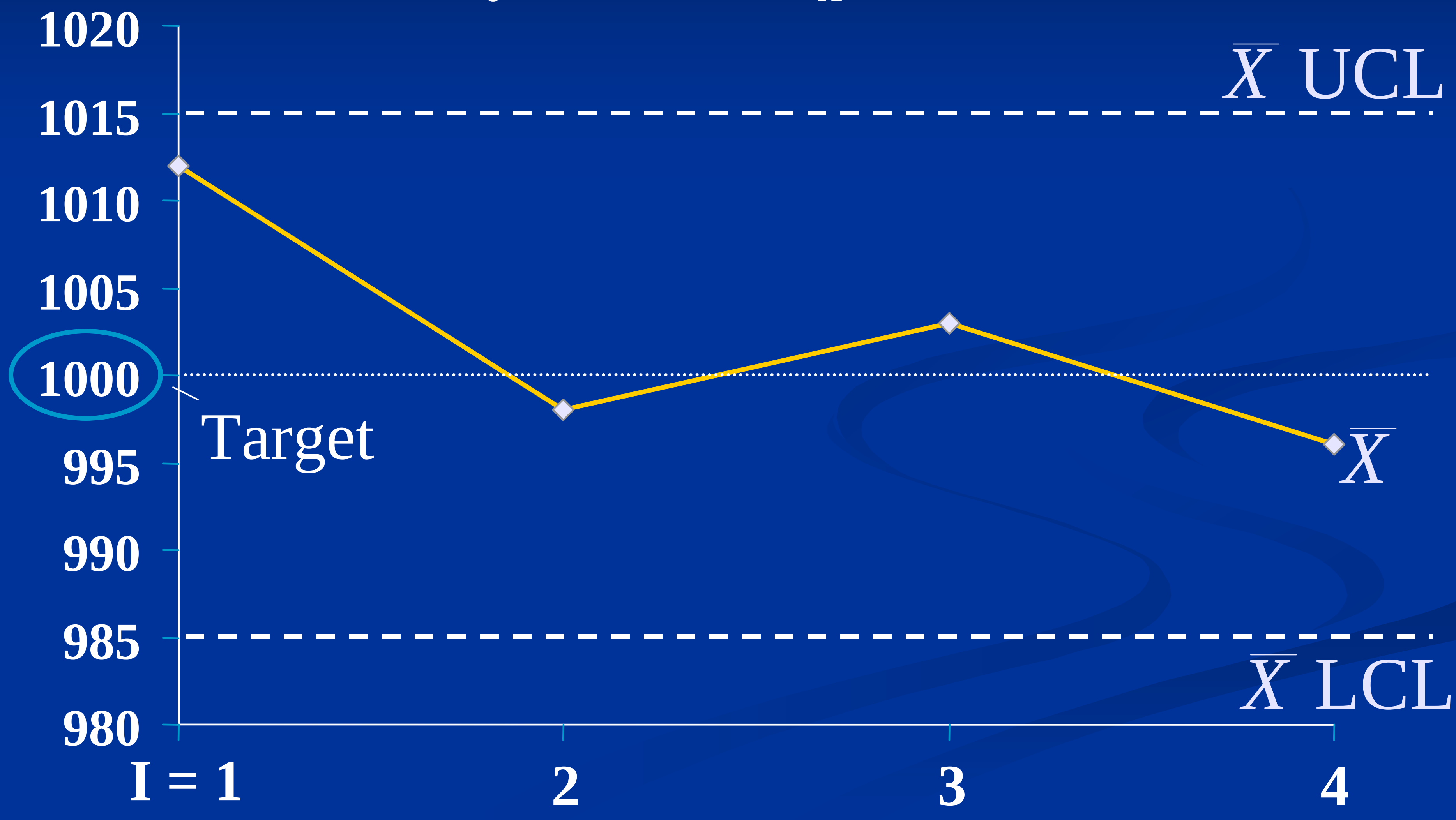
- Sample Size (**n**)
- Sampling Frequency (**h**)
- Width of Control Limits (**L**)
- "Runs Rules"

Typical X-bar Chart Design

- General Guidelines: (Duncan, 1974)
  - Samples of Size $n = 5$
  - Control limits at 3 sigma from center line
  - Sampling frequency $h = 1$ hour
- Motivations:
  - Consideration of Statistical Criteria
  - Practical Experience
  - Ease of implementation

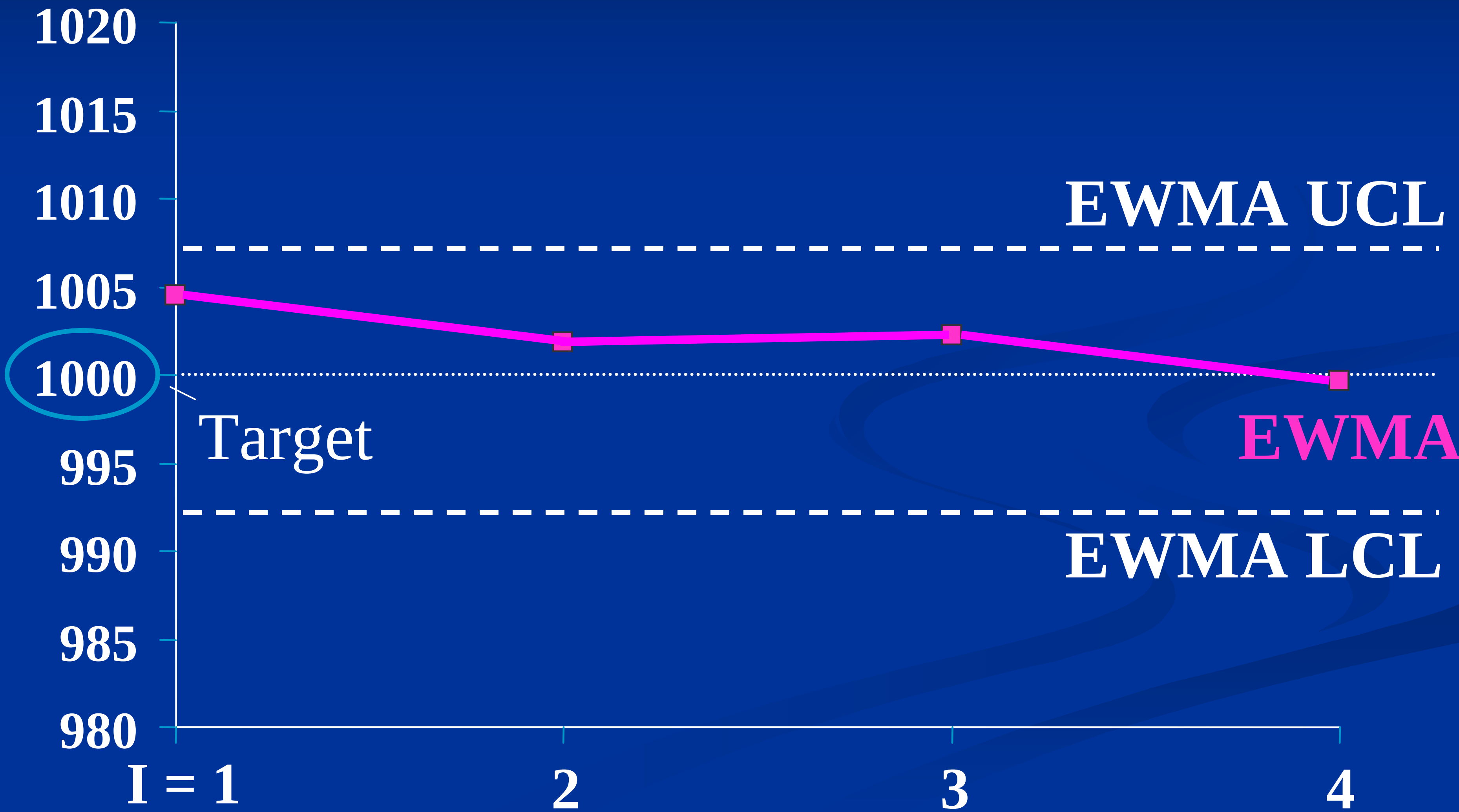
Shewhart X-bar Control Chart

$\mu_0 = 1000, \sigma_x = 5$



EWMA Control Chart

$\mu_0 = 1000, \sigma_x = 5, \sigma_z = 2.5, r = 0.4$



Exponentially Weighted Moving Average (EWMA)

$$Z_i = rX_i + (1\text{-}r)\, Z_{i-1}$$

$$Z_0 = \mu_0,\ \ 0 < r \leq 1,\ i = 1, 2, 3, \ldots$$

$$\sigma_{Z_i}^2 = \left[\frac{r\left(1-(1-r)^{2i}\right)}{2-r}\right]\sigma_X^2 \approx \left[\frac{\mathrm{r}}{2\text{-}\mathrm{r}}\right]\sigma_X^2$$

Smoothing parameter (r)
Tolerable in-control average run length (ARL)

Statistical Approach: Traditional Control Chart Design

- Usually, sample size , UCL and LCL chosen to obtain desired:
  - power of the test to detect a particular shift in the quality characteristic
  - Type I error probability
  - Sampling Interval often decided on a qualitative basis.
- **But these choices have economic implications!**

Economic Design Assumptions

- Assumptions about process
  - characterized by a single in-control state, but process can have more than one assignable cause, leading to more than one out-of-control state
  - assignable causes: Poisson process
  - Process Failure Mechanism critical
  - transitions between states instantaneous
  - process is not self-correcting

Economic Design

- Costs Considered
  - Sampling and Testing
  - Investigating Alarms
  - Repair or Correction of Assignable Causes
  - Producing defective items

Economic Design in Practice

- Practitioners aren't using economic designs. Are they? Why?
  - Models are ugly.
  - Software availability (or good simple approximations) minimal.
  - Difficulty in estimating costs.
  - Possibility of poor statistical performance, as measured by average run lengths or times to signal. Add Statistical Constraints?

Multivariate SPC Methods

- Often there are multiple related variables for defining process quality. which need to be simultaneously monitored or controlled.
- Context: Technological Improvements
  - Data-Acquisition (Automatic Inspection)
  - Process Complexity
  - Large Multivariate Data Sets of less than full rank

Requirements for a MV SPC Approach (Jackson, 1985)

- Gives a single answer to the question “Is the process under control?”
- Maintains specified Type I error probability.
- Procedure takes into account the relationships between the variables.

Problems with Multivariate SPC Charts

- Interpreting charts
- Isolating contributing variables
- Choice of design parameters
- Checking for MV Normality
- Missing data
- Detection and evaluation of the form of MV autocorrelation

MEWMA Procedure
Lowry, Woodall, Champ & Rigdon (1992)

- Effectively detects small shifts in process mean vector
- Provides forecasts of current location of process mean vector
- May be especially useful in engineering process control

The MEWMA Chart

- MEWMA Vectors are

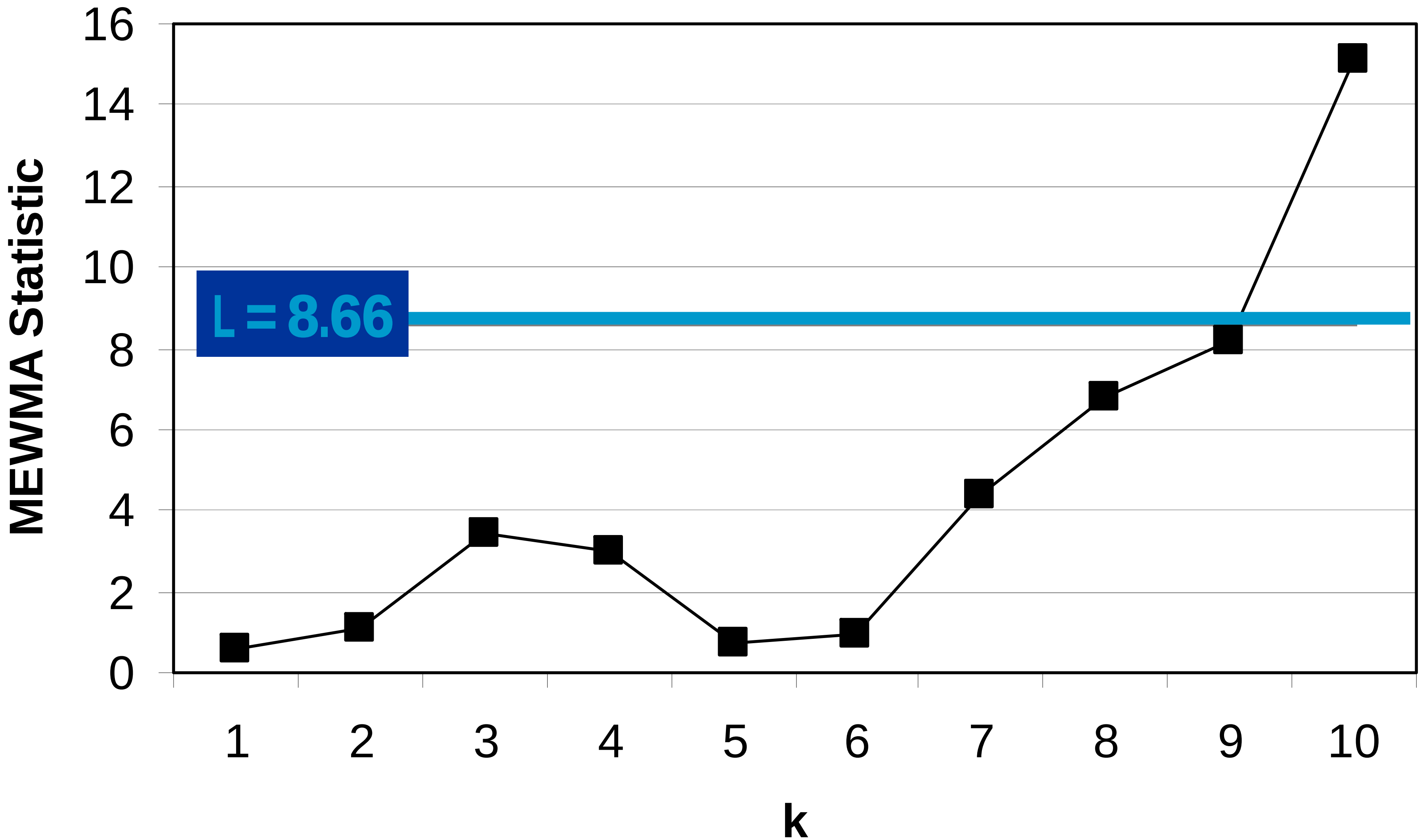
$$Z_i = R\ X_i + (I - R)\ Z_{i-1}$$

- MEWMA chart signals whenever

$$T_i^2 = \mathbf{Z_i}^{\mathrm{T}} \Sigma_{\mathbf{Z_i}}^{-1} \mathbf{Z_i} > L$$

- L chosen to achieve specified in-control ARL
- n, h, r_j are other design parameters

MEWWMA Example (LWCR)



Lorenzen-Vance Cost Model Adapted for MEWMA

- Expected Cost per Hour is

$$C(n,h,r,L) = \frac{E(C_{\text{Production}}) + E(C_{\text{Alarm\& Repair}}) + E(C_{\text{Sampling}})}{E(\text{Cycle Time})}$$

Goal: Minimize hourly production cost **C(n, h, r, L)** subject to natural restrictions on design parameters *n*, *h*, *r* and *L*.

Example 1

- P = 3 quality characteristics
- Cost parameters from univariate EWMA studies in Montgomery et al (1995):

$$\frac{1}{\lambda} = 100, E = .05, T_0 = 0, T_1 = 2, T_2 = 2$$

$$\gamma_1 = \gamma_2 = 1, C_0 = 10, C_1 = 100, Y = 50$$

$$W = 25, a = 0.5, b = 0.1$$

- Statistical constraints:

$$ARL_0 > 200,\ ARL_1 < 10$$

Detailed Results for Example 1

Economic Model (Unconstrained)

δ	n	h	r	L	Cost	ARL_0	ARL_1
0.5	15	1.52	0.63	10.51	17.80	74.9	1.94
1	14	1.90	0.93	11.76	16.72	118.5	1.30
2	9	1.80	0.90	12.40	15.96	159.5	1.15
3	7	1.63	0.98	13.85	15.65	320.3	1.12

Economic Statistical Model ($ARL_0 > 200$, $ARL_1 < 10$)

δ	n	h	r	L	Cost	ARL_0	ARL_1
0.5	17	1.39	0.56	12.71	17.91	200.7	2.08
1	14	1.73	0.92	12.85	16.77	201.5	1.40
2	10	1.78	0.93	13.95	15.97	335.7	1.15
3	7	1.63	0.96	13.81	15.65	313.6	1.12

Sensitivity of Cost to Misspecified Shift Size

Economic Design for Example 1

Assumed δ	$\delta = .5$ Cost	$\delta = 1$ Cost	$\delta = 2$ Cost	$\delta = 3$ Cost
0.5	**17.80**	16.99	16.68	16.67
1.0	18.89	**16.72**	16.29	17.00
2.0	18.27	16.19	**15.96**	15.75
3.0	29.19	19.49	16.29	**15.65**

Small cost penalty for underestimating δ.

Large cost penalty for overestimating δ.

Sensitivity of ARL_1 to Misspecified Shift Size
Economic Design for Example 1

Assumed δ	$\delta = .5$ ARL_1	$\delta = 1$ ARL_1	$\delta = 2$ ARL_1	$\delta = 3$ ARL_1
0.5	**1.94**	1.26	1.01	1.00
1.0	2.78	**1.30**	1.02	1.00
2.0	5.39	2.13	**1.15**	1.00
3.0	13.34	4.17	1.62	**1.12**

Small ARL_1 penalty for underestimating δ.
Large ARL_1 penalty for overestimating δ.

Sensitivity of Cost to Changes in Sampling Interval *h*

Economic Design for Example 1

δ	Optimal h	Resulting Cost	h = 1 Cost	h = 1.5 Cost	h = 2 Cost
0.5	**1.52**	17.80	18.11	17.80	17.93
1.0	**1.90**	16.72	17.23	16.78	16.72
2.0	**1.80**	15.96	16.30	16.00	15.97
3.0	**1.63**	15.65	15.85	15.66	15.68

Expected cost appears fairly insensitive to small changes in the value of *h*.

Do Statistical Constraints Matter?

P = 2, Mean = 100, E = .3, $\gamma_1 = \gamma_2 = 1$, $T_0 = .07$, $T_1 = .51$, $T_2 = 2.01$, $C_0 = 115.6$, $C_1 = 243.3$, Y = 195, W = 353, A = 8.75, B = 7.75, NCP = 1.14

Run	**n**	**h**	**L**	**r**	**Cost**	**ARL_0**	**ARL_1**
Ec1	5	10.37	3.16	.742	140.01	5.29	1.26
Ec2	6	9.64	4.26	.566	140.03	11.32	1.17
Ec3	5	9.32	3.90	.787	139.68	7.50	1.32
Ec4	5	9.40	3.77	.787	139.84	7.08	1.33
ES1	10	8.09	9.89	.272	148.31	190.24	1.96
ES2	10	8.30	10.23	.320	147.72	191.16	1.88

Specific Conclusions for MEWMA

- Some reasonable cost models lead to minimal impact of statistical constraints.
- Underestimating the size of shift is much safer than overestimating it.
- Cost is not especially sensitive to small changes in sampling interval parameter.
- All cost/time parameters play a role in estimating Cost of chart.
- Not all cost/time parameters relate to design parameters or ARL/ATS.

Broad Conclusions

- Large gap between methodological development and application in SPC.
- Multivariate process data can seriously benefit from multivariate control charts.
- Economic and economic-statistical designs should play a bigger role.
- Practitioners need good software for designing MEWMA charts.

Our Papers

- Linderman, Kevin and Love, Thomas E. (2000) "Economic and Economic-Statistical Designs for MEWMA Control Charts," *Journal of Quality Technology*, 32, 410-417.
- Linderman, Kevin and Love, Thomas E. (2000) "Implementing Economic and Economic-Statistical Designs for MEWMA Control Charts," *Journal of Quality Technology*, 32, 457-463.
- Love, Thomas E. & Linderman, Kevin (2003) "A Weibull Process Failure Mechanism for the Economic Design of MEWMA Control Charts" *Journal of Statistical Computation and Simulation*, *73*(3), 195-202