

# Introduction to SEM (Structural Equation Modeling)

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## Overview of Presentation

- Spend an hour on introduction to SEM
  - PART I: Where does SEM fit relative to other statistical procedures?
  - PART II: What is SEM?
    - Principles & Practice
    - Strengths & Weaknesses
- Spend half an hour looking at “Metabolic Syndrome” article
  - PART III: How to apply SEM to the study of the Metabolic Syndrome?
    - Did everyone bring a copy of the article?
    - How many of you have read the article?

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## Assumptions about Audience Participation

- I assume you will interrupt me **frequently** with questions
- Please do **not** “save” your questions for the end

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## Assumptions about Statistical Knowledge of Audience

- Neal says most of you do not know regression and factor analysis
- Logit analysis would be the common “baseline” statistic
- Also, I can assume that you all understand the concept of a correlation

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## Truth in Advertising: Caveats I

- Very difficult to do an intro to SEM in an hour and a half
- As prerequisite for SEM, I generally require at least one semester of preliminary stats
  - Regression/path analysis
  - Exploratory factor analysis

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## Truth in Advertising: Caveats II

- Building structural equation models requires “LTE”
  - Rigorous logic (“L”)
  - And deep knowledge of a substantive field’s
    - Theory (“T”)
    - Prior empirical evidence (“E”)
- I can’t apply this LTE to the “Metabolic Syndromes” article
  - I have almost no understanding of the medical terms, empirical research, or theory behind this substantive area
  - Neal and other audience members will have to supply the LTE

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## **PART I: Where does SEM fit relative to other statistical procedures?**

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## Essence of SEM

- “Full” SEM = Path analysis + Factor analysis
- “Full” SEM = Simultaneous equations + Latent variables

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## SEM Is “Superset” of Most Other Statistical Procedures

- Confirmatory and exploratory factor analysis
- Latent class analysis
- Item Response Theory (IRT)
- OLS regression
- Logit regression
- Survival (event history) analysis
- ANOVA and ANCOVA
- Repeated measures ANOVA
- Growth curve analysis
- Hierarchical Linear Modeling (Multi-level analysis, Random effects, Mixed models)

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## SEM Provides Easier Implementation of Special Statistical Procedures

- Missing values imputation (e.g., FIML)
- “Bootstrapping” of standard errors and parameter estimates for non-normally distributed data
- Monte Carlo simulations (to test “performance” of statistical procedures)
- Specification searches (for finding alternative models of equivalent “good-fit”)
- Mixture models (search for hidden “subgroups”)

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## Key References for Overview of SEM as “Superset”

- Tomarken & Waller (2005) “Strengths, Limitations & Misconceptions”
  - 2005 publication that is already “dated”
  - Which illustrates the remarkably rapid advances that occur in the SEM literature
- Muthen (2002) “Beyond SEM”
  - Simultaneous equations (econometrics)
  - Latent constructs (psychometrics)
- Neal has both articles available in PDF

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## SEM “Superset” Mentality

- Fosters a certain “arrogance” among SEM methodologists & researchers about their statistical techniques
- Later I will present a figure from Rex Kline of the “fully evolved” statistician that graphically illustrates the high opinion that SEM methodologists have of their techniques
- Given this arrogant attitude among SEM practitioners, I would argue that SEM is a “perfect match” for physicians and other researchers in field of medicine
- In other words, SEM provides statistical procedures to fit your egos!

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## SEM Software Programs

- Mplus (most all-encompassing)
- Amos (most user-friendly)
- EQS (possibly best combination of power and user-friendliness)
- Lisrel (first SEM software & still largest number of users)

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## Best Introductory Book on SEM

- Rex Kline (2005): *Principles & Practice of Structural Equation Modeling* (second edition)

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## Part II: What is SEM?

### Principles & Practice Strengths & Weaknesses

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## Principles & Practice: SEM Overview

- “Full” SEM = “Structural Model” (Path analysis) + “Measurement Model” (factor analysis)
- “Partial” SEMs
  - Path analysis (structural model) **only**
  - Factor analysis (measurement model) **only**

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## SEM Overview (cont.)

- Order of presentation
  - We'll start with Path Analysis (Structural Model)
  - Move to Factor Analysis (Measurement Model)
  - Finish with a "full" SEM

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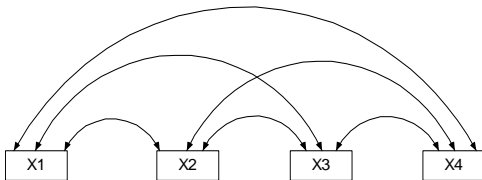
## Path Analysis ("Structural Model")

- Path analysis first developed by the geneticist: Sewell Wright
  - In other words, one of the key components of SEM came from the **medical field**
  - I'm hoping this point will add a bit of "luster" to the procedure and foster more willingness to accept it as "legitimate"
- Pure path analysis **assumes** that all observed variables (predictors and outcomes) are **measured without error**
  - i.e., No random measurement error
  - i.e., Measures have perfect "reliability"

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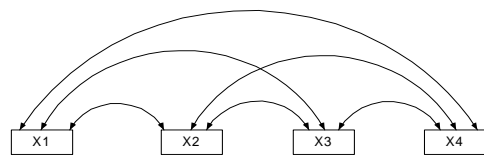
## Purpose of Path Analysis

- Path analysis attempts to **explain** "raw" **correlations** among **directly observed variables**



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## Purpose of Path Analysis (Cont.)



- "Boxes" in diagram represent directly **observed** variables
- Curved double-headed arrows represent "raw" correlations

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### Orientation of Path Analysis to More Familiar Statistical Procedures

- Logit of **direct** effects (with curved, double-headed arrows among predictors)
- OLS regression of **direct** effects (with curved, double-headed arrows among predictors)
  - Modify Amos diagram of raw correlations to display direct effects .....
- Path diagram of **direct** and **indirect** effects
  - Modify Amos diagram of direct effects to also display indirect effects .....

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### Expanding “Generic” Path Analysis to “Real-World” Data Set

- Let’s make the “X” labels in the preceding diagram concrete by using “real” variables
- Go to SPSS data matrix using “Pain,” “SOPH,” “NA” & “PA” directly observed variables .....

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### Table of Directly Observed Variables (“Boxes”)

- See Copy of Measures I handed out

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### Obtaining “Raw” Correlations (Using Amos)

- Substitute “concrete” observed variables for “generic” observed variables in Amos diagram .....
- Audience: Which correlations will be strongest?
- Run AMOS diagram to obtain raw correlations .....

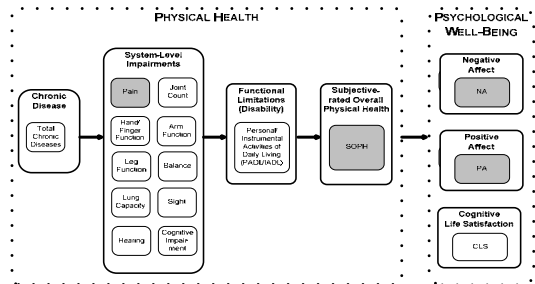
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## Explanation for these raw correlations?

- Path Analysis requires LTE
- Audience: How would you causally order set of observed variables?
- Go back to Amos diagram ....

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## “Cascade” Conceptual (LTE-Driven) Model



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## Build Path Model of Indirect & Direct Effects with Amos

- Amos Path analysis of pain, SOPH, PA & NA .....

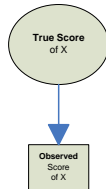
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## Reorientation: Moving from Path Analysis to Factor Analysis

- Recall SEM = Path Analysis (Structural Model) + Factor Analysis (Measurement Model)
- We will now focus on Factor Analysis
  - i.e. – Building & Analyzing the “Measurement Model”

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## Measurement Model



- Links an **observed** variable (concrete measure) to specific latent (unobserved) **construct** ("true score")
- Latent construct (a.k.a. "factor") is represented by a **circle or oval**
- Observed variable is represented by a **box**

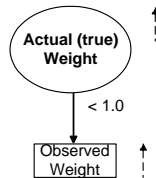
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## Latent Construct (circle)

- Latent Construct is an abstraction which may be
  - "real" or
  - "hypothetical" (possibly real)
- Examples of real constructs:
  - Weight
  - Age
- Examples of hypothetical constructs:
  - Quarks
  - Alienation

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## Example of Measurement Model for "Real" Construct (Weight)



- Note that any single, observable measure of a construct is likely to be at least a little discrepant with the true score
  - i.e., there is likely to be at least some "slippage" due to random measurement error
  - i.e., a hypothetical correlation between true score and observed score that is less than 1.0

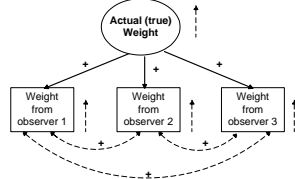
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## Multiple Indicator Measurement Model

- Multiple measures (indicators) allow assessment of "reliability"
  - i.e., assessment of "consistency" of alternative measures of a given construct
- Accordingly, one can "correct" for the degree of unreliability in measuring a given construct
  - a point I will illustrate later

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## Example of Multiple Indicator Measurement Model



- As I noted earlier, **reliability** of measures concerns the **consistency** of our (multiple) measures of a given construct
- The **higher** the **correlation** among multiple indicators of a given construct, the more consistent — i.e., **reliable** — the measures

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## Example Using SPSS with Weight Data

- We **correlate** 3 observable measures of **weight** taken on each of **five** persons:
  - i.e., sample size = 5

Subject ID	Weight 1	Weight 2	Weight 3
1	100	101	115
2	120	121	114
3	153	152	130
4	180	182	191
5	241	240	252

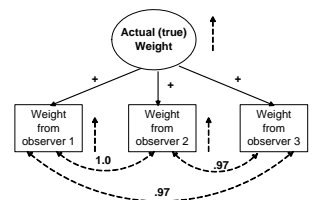
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## Example Using SPSS with Weight Data (cont.)

- NOTE: The 3 measures of **weight** could include:
  - three different observers using the same scale for each subject or
  - the same observer using three different scales for each subject
- The **major point** is that we are interested in how highly the three different measures of weight **correlate** with each other
  - i.e., how **consistent** — **reliable** — are the *three alternative measures* of the same thing
- [Go to SPSS data matrix.....]

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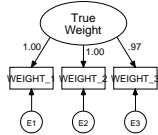
## Example Using SPSS with Weight Data (cont.)



- I have placed the SPSS correlations into our weight measurement model above
- What are the implied “factor loadings”? [Go to Amos....]

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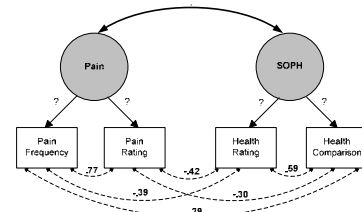
## Amos Confirmatory Factor Analysis



- What can we conclude about the reliability (consistency) of the 3 indicators of weight?
- They are so reliable we could use single indicator (box) to replace “true score”

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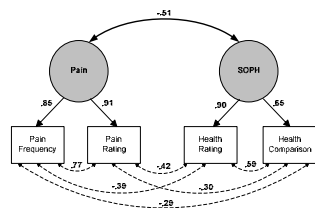
## Building a Measurement Model with More than One Construct



- I have included above the pearson correlations among indicators
- I will now run CFA with Amos to obtain factor loadings
  - [[Run Amos.....

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## Testing a Measurement Model with More than One Construct



- Note that correlation between factors is “corrected” for unreliability
  - i.e., corrected for random measurement error
- Compare this “corrected” correlation with above “observed” correlations

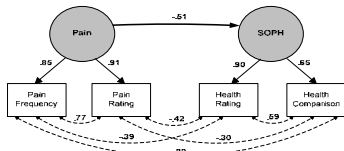
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## Reorientation: Moving from Measurement Model (Factor Analysis) to Full SEM

- Replace double-headed (“correlation”) arrows in measurement model with single-headed (“causal”) arrows
  - i.e., Add “Structural Model” (regression analysis) to “Measurement Model” (CFA)
- [[Transform Amos factor analysis drawing to Full SEM.....

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## Testing a “Full-Blown” Structural Equation Model (see Amos)



- Above figure combines structural model (regression analysis) & measurement model (CFA) [[Go to Amos.....]]
- Regression analysis provides estimate of beta coefficient (-.51) between the “true score” of Pain and the “true score” of SOPH
  - i.e., “corrected” for attenuating effects of random measurement error
- Note also that the absence of any control variables means that the Pearson correlation and the beta coefficients are identical (see prior measurement model)

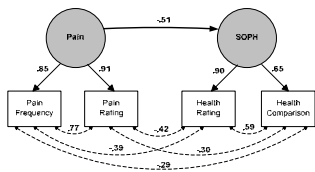
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## New Concept of “Overall Model Fit”

- Data (Correlations) = Model + Residual
  - i.e.: Observed Correlations = Model-implied correlations + Residual (difference between observed and model-implied correlations)
- Testing “Model Fit”
  - i.e.: How much “discrepancy” between observed vs. model-implied correlations?
  - Return to earlier measurement model of pain and SOPH

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## Testing Overall Model Fit



- Testing “Model Fit”
  - i.e.: How much “discrepancy” (residual) between observed correlations vs. model-implied correlations for each pair of indicators?
  - i.e., What is the “average residual” (as one overall fit index)

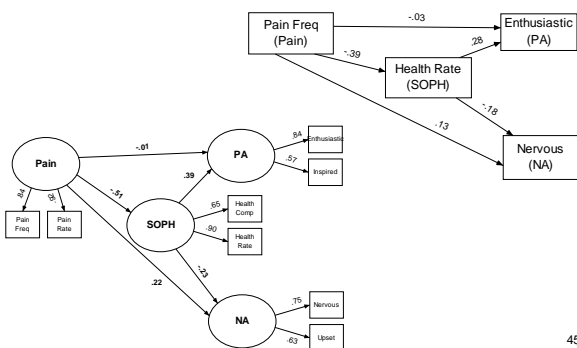
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## Overall Model Fit (cont.)

- Indices of overall fit
  - SRMR (Standardized Root Mean Square Residual) < .08 or so
  - CFI (Comparative Fit Index) > .90 to .95
  - RMSEA (Root Mean Square Error of Approximation) < .06 or so
  - Many other fit indices
- [[Go to Amos output for factor analysis of pain and SOPH to check overall model fit (e.g., CFI and RMSEA).....]]

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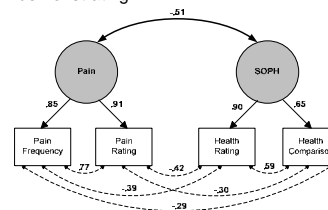
## Linking Pain, SOPH, PA, and NA: Path Analysis vs. Full SEM



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## Types of Measurement Models

- “Reflector” indicator (“effect” indicator) measurement model
  - Latent construct (factor) causes correlations among observed variables (indicators)
  - This is most common measurement model, and the one I have been demonstrating



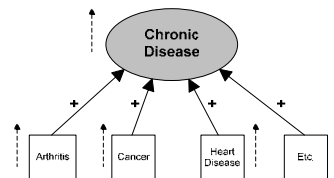
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## Types of Measurement Models (cont.)

- “Producer” indicator (“formative” indicator) measurement model
  - Observed indicators cause variation in the latent construct
  - i.e., the causal order between construct and indicators is reversed
  - Examples:
    - Chronic Diseases
    - Socioeconomic Status (SES)

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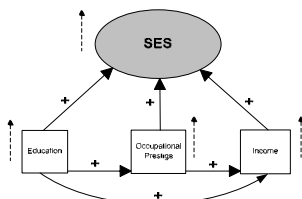
## Producer Indicator Measurement Model of Chronic Disease



- Note that each observed variable contributes to the overall score (number of chronic diseases)
- In this specific example, there would be weak correlations among indicators
  - which is contrary to what one finds with reflector indicator models

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## Producer Indicator Measurement Model of SES



- Note that this producer indicator model does display substantial correlations among indicators
- But logic suggests that increases in the observed variables are causing increases in SES, not vice versa
- Accordingly, correlations among indicators is a necessary but not sufficient condition to qualify as a reflector indicator model

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## Summary of Strengths of SEM

- Superset of diverse statistical procedures integrated into single statistical program
- Can estimate and correct for the distorting influences of both random and non-random measurement error
- Unparalleled flexibility in modeling
  - If you can envision it, SEM can test it
  - Early SEM procedures had stricter assumptions than many other statistical procedures
  - But newer SEM procedures now incorporate subset of options that, in general, allow it to make **fewer assumptions** than most alternative statistical procedures

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## Weaknesses of SEM (relative to alternative statistical procedures)

- Requires relatively large sample sizes (say, N of 150 or greater)
  - NOTE: Well-specified (good-fitting) models that have decent number of indicators (e.g., 4 or more per construct) with relatively high reliability (e.g., factor loadings of .7 or higher) may run with smaller Ns (e.g., 60 cases or less)
- Requires much more formal training in statistics to be able to effectively use SEM software programs
- Requires much more informal (“hands-on”) experience with the software programs to effectively use them
  - Extreme flexibility of SEM means there are more options to learn
  - Sophistication of estimation procedures means there are more things that can go wrong

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## Weaknesses of SEM (cont.)

- Requires well-specified (LTE-based) measurement and conceptual models
  - Because SEM is a theory-driven (confirmatory) technique, one must have well-developed a priori models
  - SEM best applied to areas of research that are “mature”
    - with well developed theories and
    - much prior empirical work
  - Very sensitive to model misspecification
    - If model is far removed from correct specification, search for correct model is not likely to be successful
    - However, new developments in “exploratory SEM” (specification searches) show promise to help find correct model even if initial model is not very close to correct model

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Earlier I noted a certain “arrogance” among SEM practitioners

- Kline’s figure of “evolved statistician”
- Go to Word document ....

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## Part III: How to Apply SEM to the Study of the Metabolic Syndrome?

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### Checklist for Metabolic Syndrome Paper (Chan et al., 2002)

- Look for two basic components
  - Measurement model (which should come first)
  - Structural model (which should come after)
- Do Structural and Measurement model make LTE-based sense?
  - Do observed variables form a reflector indicator measurement model?
  - Do latent constructs have an appropriate and clear-cut causal order?
    - Do alternative indicators of a given construct correlate relatively highly with each other relative to their correlations with indicators of other constructs?
    - Does it make logical sense that a latent construct causes these correlations among the alternative indicators?
    - Does it make logical sense that alternative indicators of a given factor will display a similar pattern of (generally weaker) correlations with other observed variables that measure other constructs (factors)

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### Checklist for Metabolic Syndrome Paper (cont.)

- Check overall model fit (assuming model actually “runs”)
  - CFI > .90 to .95
  - SRMR < .08 or so
  - RMSEA < .06 or so
  - Many other measures of overall fit
    - e.g., Chan et al. use “GFI” (Goodness of Fit Index), which should have a value of .90 to .95 or higher
- Check specific model fit
  - Are the parameter estimates (e.g., regression coefficients) in the proper (+/-) direction?
  - Is the size of the parameter estimates reasonable
    - e.g., a beta coefficient exceeding 1.0 is problematic
    - e.g., factor loadings should exceed .4 and preferably .5 or higher

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## Overview of causal model

- Go to Chan et al. PDF

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